Two-field Inflation

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Abstract

The simplest model of inflation is one in which the expansion is driven by a single scalar field. This essay will discuss the next simplest situation; two scalar fields. We will give derivations for the evolution equations of perturbations, describing their decomposition into adiabatic and entropic modes, as well as the generalisation of the slow-roll approximation. We will then examine how these extra modes affect conservation of the uniform density curvature perturbation on super-horizon scales, and the possibility of observationally distinguishing the two-field and single-field scenarios.

Contents

1 Introduction

1.1 Outline and Acknowledgments

The notation in this essay will mostly follow [1]. In this section we give an outline of two-field inflation. In section 2 we calculate the equations of motion for perturbations around the background field space trajectory, following a calculation in [2]. In section 3 we give an explanation of how the evolution of the large scale curvature depends on the adiabaticity of its initial conditions, and how adiabatic and non-adiabatic perturbations affect each other. In section 4 we present some results from [1] and explain how to link these discussions to observables, contrasting with the single field case. Finally, in section 5 we discuss non-Gaussianity in the context of two-field inflation. Using a result of [3], we present a sample calculation for an explicit potential and the curvaton scenario. In writing this essay I have found [4] and [5] very useful. I am also grateful to Dr M. C. David Marsh for helpful advice and comments.

1.2 Driving Inflation

What was the dominant component of the universe during inflation? One simple and natural answer would be a single scalar field. However, with this simplicity comes a reduction in the physics our theory can describe. For example, it turns out that for such a theory the relation between the pressure and density perturbations is homogeneous; the same equation of state is obeyed everywhere. With multiple fields we can have variations in pressure which are independent of the variations in density. As we shall see in later sections, this results in interesting new phenomenology. It also raises new challenges however, complicating analysis and making predictions sensitive to the physics of reheating.

In this essay we will consider the case where the very early universe is dominated by a pair of scalar fields. The simplest Lagrangian is the canonical[1] one:

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi) - V(\phi, \chi). \tag{1}$$

We will consider a more general case however[2], following for example [11, 1, 12]

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} G_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V(\phi^i). \tag{2}$$

[1] See e.g. [6][7][8] and references therein.

[2] A particular diagonal case is explored in [9][10].

We leave the field metric G_{ij} arbitrary, though we will assume that the curvature of the field manifold is not overly large. This adds an extra coupling between the fields, and allows the curvature of the field metric to affect the perturbations.

The desire to generate the observed approximately scale invariant power spectrum motivates what is known as the "slow-roll" approximation. This amounts to using the potential as a cosmological constant, and expanding in small parameters around perfect exponential expansion. To understand a theory of inflation we must understand the evolution of the primordial fluctuations it predicts. This means following how these evolve below, at, and above the horizon scale and determining their power spectra. The slow-roll approximation makes this calculation tractable.

It also turns out that to lowest order, we have more observables than we have slow-roll parameters; this forces the observables to obey a consistency relation that is in theory experimentally verifiable. We will see how this changes in the two-field case in section 4. Some previous works assumed that the slow-roll parameters are constant in the super-horizon limit, but various authors have shown that this is not valid; see [1] for example.

1.3 Single-field Results

Regardless of the driving force, an early period of rapid expansion results in a universe which is homogeneous, isotropic, flat, lacking in relic particles, and expanding. The mechanism of reheating is assumed to preserve these properties. Exponential expansion driven by a single slowly rolling field further predicts

- near scale invariance of primordial perturbations, with a slight red tilt;
- purely adiabatic perturbations;
- the tensor-scalar ratio and the spectral index relation $r = -8n_t$.

These predictions survive the reheating process by virtue of the fact that on large scales the curvature perturbation is constant, in the single field case. This allows us to calculate interesting quantities as they exit the horizon during inflation; they then freeze until they reenter the horizon during radiation domination, in which their evolution is well understood.

The picture is not so simple when we have more than one field. We preserve the prediction of scale invariance using the slow-roll approximation, but the perturbations are no longer purely adiabatic; it turns out that the curvature perturbation on large scales is then no longer constant. It is driven by these non-adiabatic modes (which we will call “entropic” or “isocurvature”), which can be generated by bends in the field space trajectory. If the trajectory in field space is not straight then it cannot be effectively described by a single field; isocurvature modes are generated, and we must follow the super-horizon evolution of the perturbations through to the end of inflation. The tensor fields are decoupled so n_t is still unchanged, but this new evolution causes r to vary, changing the relation above.

1.4 An Alternative

We end our introduction by mentioning another seemingly simple possibility we could consider before turning to multiple scalar fields. Vector-driven inflation was first explored in [13], however the simplest implementation of a single vector field naturally results in anisotropy. This can be avoided by considered a triplet of orthogonal vector fields, or a large number of randomly oriented ones [14], forcing us to consider multiple fields anyway.

Unlike scalar-driven inflation, the scalar, vector, and tensor modes do not necessarily decouple at linear order [15]; this complicates analysis, but does raise the possibility that such mixing may be detectable. It has also been shown that models of vector inflation are vulnerable to linear instabilities [16]; for recent work see [17].

2 Kinematics

2.1 Background Equations of Motion

We begin with the following Lagrangian, where i, j index the two fields:

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}G_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V(\phi^i). \tag{3}$$

The field metric G_{ij} is arbitrary. Specialising to an FRW spacetime metric, and assuming $\boldsymbol{\phi}$ (the vector of fields) to be homogeneous we obtain

$$\mathcal{L} = \frac{1}{2}G_{ij}\dot{\phi}^i\dot{\phi}^j - V(\phi^i), \tag{4}$$

which gives us the equation of motion:

$$\frac{D}{dt}\dot{\boldsymbol{\phi}} + 3H\dot{\boldsymbol{\phi}} = -\boldsymbol{\nabla}^\dagger V. \tag{5}$$

The background fields are coupled not only through the potential, but also through the covariant derivative.

We can recast this in terms of a dimensionless parameter, namely the number of e-folds:

$$\begin{aligned} &dN \equiv d\ln a \\ \Longrightarrow\ &\frac{D\boldsymbol{\phi}}{dN} \equiv \boldsymbol{\phi}' = \frac{\dot{\boldsymbol{\phi}}}{H} \\ \Longrightarrow\ &H^2\boldsymbol{\phi}'' + \left(3H^2 + H'H\right)\boldsymbol{\phi}' = -\boldsymbol{\nabla}^\dagger V. \end{aligned} \tag{6}$$

In general we will use $'$ to refer to the covariant derivative with respect to e-folds. Following the notation of [1] we will now define two dimensionless quantities which we will use in our slow-roll approximation. We want near exponential expansion to achieve the observed near scale invariance of the power spectra; motivated by this, we define $\epsilon \equiv -(\ln H)'$, the usual slow-roll parameter. We further define $\boldsymbol{\eta} \equiv \frac{D\boldsymbol{\phi}'}{dN}$, the covariant acceleration. This reduces to $\frac{\dot{\phi}}{H}\left(\frac{\ddot{\phi}}{H\dot{\phi}} + \epsilon\right)$ in the single field case, and so is small when ϵ is small and $\ddot{\phi} \ll H\dot{\phi}$.

Consider the Friedmann equation, with $v \equiv |\boldsymbol{\phi}'|$:

$$\begin{aligned} H^2 &= \frac{1}{3}\left(\frac{1}{2}v^2H^2 + V\right) \\ \Longrightarrow\ H^2 &= \frac{V}{3 - \frac{1}{2}v^2}. \end{aligned} \tag{7}$$

Taking the logarithmic derivative, we get

$$
\begin{aligned}
-2\epsilon &= \boldsymbol{\phi}' \cdot \boldsymbol{\nabla}^\dagger \ln V + \frac{vv'}{3 - \frac{1}{2}v^2} \\
&= \boldsymbol{\phi}' \cdot \boldsymbol{\nabla}^\dagger \ln V + \frac{H^2}{V}\boldsymbol{\phi}' \cdot \boldsymbol{\phi}'' \\
&= \boldsymbol{\phi}' \cdot \boldsymbol{\nabla}^\dagger \ln V - \frac{1}{V}\boldsymbol{\phi}' \cdot \left(H^2 \left(3-\epsilon\right)\boldsymbol{\phi}' + \boldsymbol{\nabla}^\dagger V\right) \\
&= -\frac{3-\epsilon}{3 - \frac{1}{2}v^2}(v^2) \\
\implies \epsilon &= \frac{1}{2}v^2
\end{aligned}
\tag{8}
$$

so ϵ measures the magnitude of the velocity of the field vector.

Using this in our equation of motion and Friedmann equation, we obtain [1]

$$
\frac{\boldsymbol{\eta}}{3-\epsilon} + \boldsymbol{\phi}' = -\boldsymbol{\nabla}^\dagger \ln V, \tag{9}
$$

$$
H^2 = \frac{V}{3-\epsilon}. \tag{10}
$$

Note that these equations are exact; no approximation has yet been made.

2.2 Slow-Roll Conditions for Multi-Field Inflation

The slow-roll conditions presented in [1] are motivated by the relation $V = H^2\left(3-\epsilon\right)$; we wish for V to act as a cosmological constant, so we impose

$$
|(\ln H)'| \ll 1, \tag{11}
$$

$$
|(\ln \epsilon)'| \ll 1, \tag{12}
$$

since the product of two slowly varying quantities is also slowly varying. Note that the first condition is simply that $\epsilon \ll 1$, the usual slow-roll condition. Using that $\epsilon = \frac{1}{2}\boldsymbol{\phi}' \cdot \boldsymbol{\phi}'$, we find that the second condition is equivalent to

$$
\left|\frac{\eta_\parallel}{v}\right| \ll 1 \tag{13}
$$

where $\eta_\parallel$ is the component of the acceleration of the field vector along the trajectory (so along $\boldsymbol{\phi}'$).[3]

[3] To be explicit $\frac{\eta_\parallel}{v} = \frac{1}{2}(\ln\epsilon)'$; some authors define $\eta \equiv (\ln\epsilon)'$.

This second slow-roll condition is weaker than the usual assumption:

$$\begin{aligned} &\left|\ddot{\boldsymbol{\phi}}\right| \ll 3H\left|\dot{\boldsymbol{\phi}}\right| \\ \implies &\left|H(H'\boldsymbol{\phi}' + H\boldsymbol{\phi}'')\right| \ll 3H^2\left|\boldsymbol{\phi}'\right| \\ \implies &\left|\boldsymbol{\eta} - \boldsymbol{\phi}'\epsilon\right| \ll 3v \end{aligned} \tag{14}$$

Using the reverse triangle inequality, we obtain

$$\begin{aligned} &|\boldsymbol{\eta}| = |\boldsymbol{\eta} - \boldsymbol{\phi}'\epsilon + \boldsymbol{\phi}'\epsilon| \\ \implies &|\boldsymbol{\eta}| < |\boldsymbol{\eta} - \boldsymbol{\phi}'\epsilon| + \frac{1}{2}v^3 \\ \implies &|\boldsymbol{\eta}| \ll 3v \end{aligned} \tag{15}$$

if ϵ is small. This is a constraint on the whole vector $\boldsymbol{\eta}$, as opposed to the constraint on the component parallel to the trajectory above. Constraining $\frac{\eta_\perp}{v}$ to be small is called the "slow-turn" approximation; we will see later that the behaviour of $\frac{\eta_\parallel}{v}$ and $\frac{\eta_\perp}{v}$ affect the perturbations in different ways, so it is useful to make the split explicit.

In the slow-roll and slow-turn approximation (9) reduces to $\boldsymbol{\phi}' = -\boldsymbol{\nabla}^\dagger \ln V$ which means that $|\boldsymbol{\nabla} \ln V|^2$ is order ϵ. Taking the derivative of (9), noting that $\frac{D}{dN} = v\nabla_\parallel$, we get

$$\nabla_\parallel\nabla_\parallel \ln V = -\frac{\eta_\parallel}{v} - \frac{3+\epsilon}{(3-\epsilon)^2}\left(\frac{\eta_\parallel}{v}\right)^2 + \frac{1}{3-\epsilon}\left(\frac{\eta_\perp}{v}\right)^2 - \frac{1}{3-\epsilon}\left(\frac{\eta_\parallel}{v}\right)', \tag{16}$$

$$\nabla_\parallel\nabla_\perp \ln V = -\frac{\eta_\perp}{v} - \frac{6}{(3-\epsilon)^2}\frac{\eta_\parallel}{v}\frac{\eta_\perp}{v} - \frac{1}{3-\epsilon}\left(\frac{\eta_\perp}{v}\right)', \tag{17}$$

placing slow-roll and slow-turn constraints on those second derivatives of $\ln V$.

2.3 Perturbations about the Background

Here we derive the equations governing the evolution of the perturbations. We continue to use the language of [1], using dimensionless quantites and keeping the slow-roll and slow-turn parameters explicit but making no approximations as yet. We present a calculation from [2] in this language.

We begin with the full equation of motion:

$$\begin{aligned} &(D_\mu + \partial_\mu(\ln\sqrt{-g}))g^{\mu\nu}\partial_\nu\boldsymbol{\phi} + \boldsymbol{\nabla}^\dagger V = 0 \\ \implies &D_\mu(g^{\mu\nu})\partial_\nu\boldsymbol{\phi} + g^{\mu\nu}D_\mu(\partial_\nu\boldsymbol{\phi}) + \partial_\mu(\ln\sqrt{-g})g^{\mu\nu}\partial_\nu\boldsymbol{\phi} + \boldsymbol{\nabla}^\dagger V = 0. \end{aligned} \tag{18}$$

We will take the first order variation, choosing our gauge so that the metric looks like

$$ds^2 = (1 + 2A)dt^2 - a^2[\delta_{ij} + 2\partial_i\partial_j E]dx^i dx^j \tag{19}$$

which gives us $\delta(\ln\sqrt{-g}) = A - k^2E$. We find:

$$\begin{aligned} H^2\delta(\boldsymbol{\phi}'') + (3H^2 + HH')\delta(\boldsymbol{\phi}') + \frac{k^2}{a^2}\boldsymbol{\delta\phi} + \boldsymbol{\delta\phi}\cdot\boldsymbol{\nabla}(\boldsymbol{\nabla}^\dagger V) \\ = \boldsymbol{\phi}''(2AH^2) + \boldsymbol{\phi}'(2A(3H^2 + HH') + H^2(A' + k^2E')). \end{aligned} \tag{20}$$

Using (10) and $\epsilon = -\frac{H'}{H}$ we find $3H^2 + HH' = V$:

$$\begin{aligned} H^2\delta(\boldsymbol{\phi}'') + V\delta(\boldsymbol{\phi}') + \frac{k^2}{a^2}\boldsymbol{\delta\phi} + \boldsymbol{\delta\phi}\cdot\boldsymbol{\nabla}(\boldsymbol{\nabla}^\dagger V) \\ = \boldsymbol{\phi}''(2AH^2) + \boldsymbol{\phi}'(2AV + H^2(A' + k^2E')). \end{aligned} \tag{21}$$

$\boldsymbol{\delta\phi}$ is Lie transported along the trajectory so $\delta(\boldsymbol{\phi}') = (\boldsymbol{\delta\phi})'$:

$$\begin{aligned} \delta(\boldsymbol{\phi}'') &= (\delta D_N D_N - D_N\delta D_N + D_N D_N\delta)\boldsymbol{\phi} \\ &= (\delta D_N - D_N\delta)\boldsymbol{\phi}' + D_N D_N\boldsymbol{\delta\phi} \\ &= ((\boldsymbol{\delta\phi}^\dagger\boldsymbol{\nabla})(\boldsymbol{\phi}'^\dagger\boldsymbol{\nabla}) - (\boldsymbol{\phi}'^\dagger\boldsymbol{\nabla})(\boldsymbol{\delta\phi}^\dagger\boldsymbol{\nabla}))\boldsymbol{\phi}' + (\boldsymbol{\delta\phi})'' \\ &= \boldsymbol{R}(\boldsymbol{\delta\phi}, \boldsymbol{\phi}')\boldsymbol{\phi}' + (\boldsymbol{\delta\phi})'' \end{aligned} \tag{22}$$

where $\boldsymbol{R}$ is the Riemann curvature tensor[4]. But, since we are in two dimensions, the Bianchi identites imply that $R_{abcd} = \frac{1}{2}R(G_{ac}G_{bd} - G_{ad}G_{bc})$, so:

$$\begin{aligned} R_{abcd}\delta\phi^c\phi'^b\phi'^d &= \frac{1}{2}R(G_{ac}G_{bd} - G_{ad}G_{bc})\delta\phi^c\phi'^b\phi'^d \\ &= \delta\phi_b\epsilon R\left(\delta^b_a - \frac{\phi'^b\phi'_a}{\phi'_c\phi'^c}\right) \\ &= \delta\phi_b\epsilon R\left(\delta^b_a - \boldsymbol{e}^b_\parallel\boldsymbol{e}_{\parallel a}\right) \\ &= \delta\phi_b\epsilon R\boldsymbol{e}^b_\perp\boldsymbol{e}_{\perp a} \end{aligned} \tag{23}$$

where we have defined the unit vector along the trajectory $\boldsymbol{e}_\parallel = \frac{\boldsymbol{\phi}'}{\sqrt{\boldsymbol{\phi}'\cdot\boldsymbol{\phi}'}}$. From the perturbed Einstein equations we have that $2A = \boldsymbol{\phi}'\cdot\boldsymbol{\delta\phi} = v\delta\phi_\parallel$; note

[4]Note that [1] uses a non-standard sign convention for $\boldsymbol{R}$ but not for R^a_{bcd}.

that this means the component of the perturbation along the background trajectory corresponds to a local time shift in this gauge. We also have $A' + k^2 E' = \boldsymbol{\delta\phi} \cdot \boldsymbol{\phi}''$, so

$$\begin{aligned} H^2(\boldsymbol{\delta\phi})'' + V(\boldsymbol{\delta\phi})' + \frac{k^2}{a^2}\boldsymbol{\delta\phi} + \boldsymbol{\delta\phi} \cdot \boldsymbol{\nabla}(\boldsymbol{\nabla}^\dagger V) + H^2 \boldsymbol{\delta\phi} \epsilon R \boldsymbol{e}_\perp \boldsymbol{e}_\perp^\dagger \\ = \boldsymbol{\phi}''\boldsymbol{\phi}' \cdot \boldsymbol{\delta\phi} H^2 + \boldsymbol{\phi}'(\boldsymbol{\phi}' \cdot \boldsymbol{\delta\phi} V + H^2 \boldsymbol{\delta\phi} \cdot \boldsymbol{\phi}''). \end{aligned} \tag{24}$$

Substituting (10):

$$\begin{aligned} \frac{V}{3-\epsilon}(\boldsymbol{\delta\phi})'' + V(\boldsymbol{\delta\phi})' + \frac{k^2}{a^2}\boldsymbol{\delta\phi} + \boldsymbol{\delta\phi} \cdot \boldsymbol{\nabla}(\boldsymbol{\nabla}^\dagger V) + \frac{V}{3-\epsilon}\boldsymbol{\delta\phi} \epsilon R \boldsymbol{e}_\perp \boldsymbol{e}_\perp^\dagger \\ = \boldsymbol{\phi}''\boldsymbol{\phi}' \cdot \boldsymbol{\delta\phi}\frac{V}{3-\epsilon} + \boldsymbol{\phi}'\boldsymbol{\phi}' \cdot \boldsymbol{\delta\phi} V + \boldsymbol{\phi}'\frac{V}{3-\epsilon}\boldsymbol{\delta\phi} \cdot \boldsymbol{\phi}'' \\ = V\left(\boldsymbol{\phi}''\boldsymbol{\phi}'^\dagger \frac{1}{3-\epsilon} + \boldsymbol{\phi}'\boldsymbol{\phi}'^\dagger + \boldsymbol{\phi}'\boldsymbol{\phi}''^\dagger \frac{1}{3-\epsilon}\right)\boldsymbol{\delta\phi} \\ = V\left(\left(\boldsymbol{\phi}' + \frac{\boldsymbol{\phi}''}{3-\epsilon}\right)\left(\boldsymbol{\phi}'^\dagger + \frac{\boldsymbol{\phi}''^\dagger}{3-\epsilon}\right) - \frac{\boldsymbol{\phi}''\boldsymbol{\phi}''^\dagger}{(3-\epsilon)^2}\right)\boldsymbol{\delta\phi} \\ = V\left(\boldsymbol{\nabla}^\dagger \ln V \boldsymbol{\nabla} \ln V - \frac{\boldsymbol{\phi}''\boldsymbol{\phi}''^\dagger}{(3-\epsilon)^2}\right)\boldsymbol{\delta\phi} \end{aligned} \tag{25}$$

where we used the equation of motion (9).

Consider $\boldsymbol{\nabla}^\dagger \boldsymbol{\nabla} V$:

$$\begin{aligned} \frac{\boldsymbol{\nabla}^\dagger \boldsymbol{\nabla} V}{V} &= \frac{\boldsymbol{\nabla}^\dagger \boldsymbol{\nabla} V}{V} - \frac{1}{V^2}\boldsymbol{\nabla}^\dagger V \boldsymbol{\nabla} V + \frac{1}{V^2}\boldsymbol{\nabla}^\dagger V \boldsymbol{\nabla} V \\ &= \boldsymbol{\nabla}^\dagger \boldsymbol{\nabla}(\ln V) + \boldsymbol{\nabla}^\dagger \ln V \boldsymbol{\nabla} \ln V \\ &= \boldsymbol{M} + \boldsymbol{\nabla}^\dagger \ln V \boldsymbol{\nabla} \ln V \end{aligned} \tag{26}$$

where we defined $\boldsymbol{M}$ as $\boldsymbol{\nabla}^\dagger \boldsymbol{\nabla}(\ln V)$.

So, finally, we get

$$\frac{1}{3-\epsilon}(\boldsymbol{\delta\phi})'' + (\boldsymbol{\delta\phi})' + \frac{k^2}{a^2 V}\boldsymbol{\delta\phi} = -\left(\boldsymbol{M} + \frac{\boldsymbol{\eta}\boldsymbol{\eta}^\dagger}{(3-\epsilon)^2} + \frac{1}{3-\epsilon}\epsilon R \boldsymbol{e}_\perp \boldsymbol{e}_\perp^\dagger\right)\boldsymbol{\delta\phi}. \tag{27}$$

This can be decomposed in components parallel and perpendicular to the trajectory ($\boldsymbol{\delta\phi} = \delta\phi_\parallel \boldsymbol{e}_\parallel + \delta\phi_\perp \boldsymbol{e}_\perp$), corresponding to adiabatic and entropic perturbations respectively. Note that

$$\begin{aligned}
\boldsymbol{e}_{\parallel}' &= \frac{\phi_1''\phi_2' - \phi_2''\phi_1'}{v^2}\boldsymbol{e}_{\perp} \\
&= \frac{\boldsymbol{\eta}\cdot\boldsymbol{e}_{\perp}}{v}\boldsymbol{e}_{\perp} \\
&= \frac{\eta_{\perp}}{v}\boldsymbol{e}_{\perp}
\end{aligned} \tag{28}$$

and similarly $\boldsymbol{e}_{\perp}' = -\frac{\eta_{\perp}}{v}\boldsymbol{e}_{\parallel}$. We call $\frac{\eta_{\perp}}{v}$ the turn rate. Following [1] we split this into components, using (16), (17) and $\epsilon = \frac{1}{2}v^2$:

$$\begin{aligned}
\frac{1}{3-\epsilon}\delta\phi_{\perp}'' + \delta\phi_{\perp}' + \left(\frac{k^2}{a^2V}\right)\delta\phi_{\perp} + \left[M_{\perp\perp} + \frac{\epsilon R}{3-\epsilon} - \frac{3(1-\epsilon)}{(3-\epsilon)^2}\left(\frac{\eta_{\perp}}{v}\right)^2\right]\delta\phi_{\perp} \\
= -\frac{2}{3-\epsilon}\left(\frac{\eta_{\perp}}{v}\right)\left[\delta\phi_{\parallel}' - \frac{\eta_{\parallel}}{v}\delta\phi_{\parallel}\right],
\end{aligned} \tag{29}$$

$$\begin{aligned}
\frac{1}{3-\epsilon}\delta\phi_{\parallel}'' + \delta\phi_{\parallel}' + \left(\frac{k^2}{a^2V}\right)\delta\phi_{\parallel} - \frac{1}{3-\epsilon}\left[(3-\epsilon)\frac{\eta_{\parallel}}{v} + \left(\frac{\eta_{\parallel}}{v}\right)^2 + \left(\frac{\eta_{\parallel}}{v}\right)'\right]\delta\phi_{\parallel} \\
= 2\delta\phi_{\perp}\left[\frac{\eta_{\perp}}{v} + \frac{1}{3-\epsilon}\left(\left(\frac{\eta_{\perp}}{v}\right)' + \frac{\eta_{\parallel}}{v}\frac{\eta_{\perp}}{v}\right)\right] + \frac{2\delta\phi_{\perp}'}{3-\epsilon}\frac{\eta_{\perp}}{v}.
\end{aligned} \tag{30}$$

Note that the turn rate of the background trajectory in field space controls the coupling between the two modes.

From [1], the second equation can be rewritten as

$$\left[3-\epsilon+\frac{\eta_{\parallel}}{v}+\frac{D}{dN}\right]\left(\delta\phi_{\parallel}' - \frac{\eta_{\parallel}}{v}\delta\phi_{\parallel} - 2\frac{\eta_{\perp}}{v}\delta\phi_{\perp}\right) = 0 \tag{31}$$

where we have dropped a term negligible at super-horizon scales. The solution has two modes; one is decaying during slow-roll, the other is constant:

$$\delta\phi_{\parallel}' - \frac{\eta_{\parallel}}{v}\delta\phi_{\parallel} - 2\frac{\eta_{\perp}}{v}\delta\phi_{\perp} = 0. \tag{32}$$

We see that for a straight trajectory $\delta\phi_{\parallel} \propto v$, but during a turn the adiabatic modes are sourced by the entropic modes. Using this, we get

$$\frac{1}{3-\epsilon}\delta\phi_{\perp}'' + \delta\phi_{\perp}' + \left[\left(\frac{k^2}{a^2V}\right) + M_{\perp\perp} + \frac{\epsilon R}{3-\epsilon} + \frac{9-\epsilon}{(3-\epsilon)^2}\left(\frac{\eta_{\perp}}{v}\right)^2\right]\delta\phi_{\perp} = 0. \tag{33}$$

Note that the entropic modes evolve independently of the adiabatic modes, greatly simplifying their solutions. The Hessian of the potential, the curvature of the field manifold and the turn rate all contribute to the entropic mass.

For a field manifold with $|R|$ of order 1 the curvature contribution is small during slow roll. However, as discussed in [1, 18], it becomes significant at the end of inflation as ϵ is no longer negligible. $R > 0$ acts to stabilise the perturbations, but $R < 0$ can result in a tachyonic entropic mass. This is intuitive when thought of in terms of the convergence or divergence of the ensemble of perturbed trajectories.

The contribution from the turn rate is always positive; during a turn (exactly when it sources the adiabatic perturbations) it acts to damp the entropic perturbations. From these considerations we can infer the behaviour of the entropic modes from the background kinematics.

3 Entropic Modes

3.1 Separate Universe Assumption

Here we mostly follow [19]. We assume that there is some scale sufficiently large that we can neglect gradients. This means that each region in the perturbed universe evolves like an unperturbed FRW universe, but with different parameters. We consider the uniform density curvature perturbation $\mathcal{R}$ on scales large enough that this separate universe picture is accurate.

3.2 Effectively Single Field Trajectories

Adiabatic initial conditions imply that P is determined uniquely by ρ; they obey the same equation of state throughout the universe. In turn, ρ is determined by the integrated expansion, N. This means that the perturbations can be described entirely as a local time shift. Each region undergoes the same evolution, displaced slightly *along*, but not off, the same FRW trajectory. The separate universe assumption then implies two things:

1. If the perturbations are initially adiabatic, they will remain adiabatic [20]. This is reflected in (33).

2. The integrated expansion that a region undergoes between two uniform density hypersurfaces does not vary from region to region.

The first point implies that at these scales adiabatic perturbations cannot source entropy perturbations. The second, that for large wavelength perturbations with adiabatic initial conditions, $\mathcal{R}$ is constant.

3.3 Multi-Field Effects, Evolution of $\mathcal{R}$

The picture is different when we allow multi-field effects, which can set up perturbations with different equations of state in different regions. In this case we have entropy perturbations, off the background trajectory. It is no longer true that all regions follow the same evolution, displaced by a local time shift. The integrated expansion between uniform density hypersurfaces now varies between regions. This not only generates further adiabatic and entropy perturbations, but also causes non-neglibile evolution in $\mathcal{R}$, even on scales much larger than the horizon.

We can define adiabatic perturbation as those which satisfy $\frac{\delta P}{P'} = \frac{\delta \rho}{\rho'}$. Note that this is always true when P is uniquely determined by ρ; i.e. P and ρ

obey the same equation of state everywhere. In general, we must also include non-adiabatic perturbations, which we can define as

$$\delta P_{nad} \equiv \delta P - \frac{P'}{\rho'}\delta\rho. \tag{34}$$

These are perturbations which affect P and ρ in such a way that each region obeys it own equation of state; for example, if $\delta\rho$ was zero everywhere, but δP still varied. Recalling that under $N \to N + \delta N$

$$\delta\rho \to \delta\rho - \rho'\delta N, \tag{35}$$

$$\delta P \to \delta P - P'\delta N. \tag{36}$$

we can see explicitly that for adiabatic perturbations ($\delta P_{nad} = 0$) surfaces of uniform pressure coincide with surfaces of uniform density (note that uniform density corresponds to vanishing *adiabatic* perturbations).

The same goes for $\boldsymbol{\phi}$:

$$\boldsymbol{\delta\phi} \to \boldsymbol{\delta\phi} - \boldsymbol{\phi}'\delta N. \tag{37}$$

If $\boldsymbol{\delta\phi}$ is defined on a spatially flat hypersurface, then the separation between that surface and one of uniform density obeys

$$\begin{aligned} \delta\phi_{\parallel} &= v\delta N \\ \implies \delta N &= \frac{\delta\phi_{\parallel}}{v}. \end{aligned} \tag{38}$$

The δN formalism[5] implies that $\mathcal{R} = \delta N$; we then have

$$\mathcal{R} = \frac{\delta\phi_{\parallel}}{v}. \tag{39}$$

It is then natural to define the isocurvature perturbations as

$$\mathcal{S} \equiv \frac{\delta\phi_{\perp}}{v}. \tag{40}$$

We can calculate

$$\mathcal{R}' = 2\left(\frac{\eta_{\perp}}{v}\right)\mathcal{S}, \tag{41}$$

again using (32).

[5] I.e. it is the difference in integrated expansion that causes curvature perturbations. See, for example, [11].

4 Connection to Observables

4.1 Power Spectra

Quantizing (27), it is shown in [1] that while to lowest order in slow-roll the fluctuations in the scalar fields are independent and Gaussian, they are correlated at first order. Defining $\tilde{\boldsymbol{M}} \equiv \boldsymbol{M} + \frac{\epsilon R}{3-\epsilon}\boldsymbol{e}_\perp \boldsymbol{e}_\perp^\dagger$, the results for the adiabatic power spectrum, isocurvature power spectrum and cross correlation between the two are:

$$\mathcal{P}^*_{\mathcal{R}} = \left(\frac{H}{2\pi v}\right)^2 (1 + 2(C-1)\epsilon - 2C\tilde{M}_{\parallel\parallel}), \tag{42}$$

$$\mathcal{P}^*_{\mathcal{S}} = \left(\frac{H}{2\pi v}\right)^2 (1 + 2(C-1)\epsilon - 2C\tilde{M}_{\perp\perp}), \tag{43}$$

$$\mathcal{C}^*_{\mathcal{R}\mathcal{S}} = \left(\frac{H}{2\pi v}\right)^2 (-2C\tilde{M}_{\parallel\perp}). \tag{44}$$

The right hand side of each equality is evaluated at Hubble crossing ($*$ denotes the value at Hubble crossing). $C = 2 - \ln 2 - \gamma \approx 0.7296$, where γ is the Euler-Mascheroni constant; this comes from an asymptotic expansion of the Hankel function solution of the quantized equations of motion. We can see from (17) that the cross-correlation (in the absence of field curvature) is controlled by the turn rate; if the background trajectory is turning as a scale crosses the horizon the isocurvature and adiabatic perturbations will be correlated.

The presence of isocurvature perturbations now complicates the story compared to the single-field case. $\mathcal{P}^*_{\mathcal{R}}$ now evolves outside the horizon; we cannot ignore the details of the physics between horizon exit and the start of radiation domination. The transfer function formalism of [6] allows us to parametrise our ignorance of this period. From the considerations in 3, we know that both $\mathcal{R}$ and $\mathcal{S}$ are sourced only by $\mathcal{S}$. So, we can describe their large scale evolution with some functions α and β:

$$\mathcal{R}' = \alpha\mathcal{S}, \tag{45}$$

$$\mathcal{S}' = \beta\mathcal{S}. \tag{46}$$

Integrating these from horizon exit (which is scale dependent) to the end of inflation, we can write the transfer functions:

$$\mathcal{R} = \mathcal{R}^* + T_{\mathcal{R}\mathcal{S}}\mathcal{S}^*, \tag{47}$$

$$\mathcal{S} = T_{\mathcal{S}\mathcal{S}}\mathcal{S}^*. \tag{48}$$

This gives us the power spectra at the end of inflation to lowest order in slow-roll:

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi v}\right)^2 (1 + T_{\mathcal{RS}}^2), \tag{49}$$

$$\mathcal{C}_{\mathcal{RS}} = \left(\frac{H}{2\pi v}\right)^2 (T_{\mathcal{RS}} T_{\mathcal{SS}}), \tag{50}$$

$$\mathcal{P}_{\mathcal{S}} = \left(\frac{H}{2\pi v}\right)^2 (T_{\mathcal{SS}}^2). \tag{51}$$

H and v are evaluated at horizon exit, and the transfer functions are evaluated at the end of inflation.

The gravitational fluctuations are decoupled from the scalar fluctuations at linear order, so do not evolve at large scales regardless of the number of fields.

$$\mathcal{P}_T = 8\left(\frac{H}{2\pi}\right)^2, \tag{52}$$

$$n_T = -2\epsilon. \tag{53}$$

As in [1] we define the correlation angle as

$$\sin\Delta_N \equiv \frac{T_{\mathcal{RS}}}{\sqrt{1 + T_{\mathcal{RS}}^2}} \approx \frac{\mathcal{C}_{\mathcal{RS}}}{\sqrt{\mathcal{P}_{\mathcal{R}}\mathcal{P}_{\mathcal{S}}}}. \tag{54}$$

Defining $\boldsymbol{e}_N = \cos\Delta_N \boldsymbol{e}_{\parallel} + \sin\Delta_N \boldsymbol{e}_{\perp}$, the authors of [1] find the spectral indices to first order:

$$n_{\mathcal{R}} = -2\epsilon + 2\boldsymbol{e}_N^{\dagger}\tilde{\boldsymbol{M}}\boldsymbol{e}_N, \tag{55}$$

$$n_{\mathcal{C}} = -2\epsilon + 2\boldsymbol{e}_N^{\dagger}\tilde{\boldsymbol{M}}\boldsymbol{e}_{\perp}\sin^{-1}\Delta_N, \tag{56}$$

$$n_{\mathcal{S}} = -2\epsilon + 2\boldsymbol{e}_{\perp}^{\dagger}\tilde{\boldsymbol{M}}\boldsymbol{e}_{\perp}. \tag{57}$$

Through $\boldsymbol{e}_N$, $n_{\mathcal{R}}$ has a much stronger dependence on $T_{\mathcal{RS}}$ than $n_{\mathcal{S}}$ does; note also that in $n_{\mathcal{C}}$ there is scope for significant scale dependence. Recall the term $\frac{\epsilon R}{3-\epsilon}\boldsymbol{e}_{\perp}\boldsymbol{e}_{\perp}^{\dagger}$ in $\tilde{\boldsymbol{M}}$. We see that the dependence of $n_{\mathcal{R}}$ on the curvature of the field manifold is directly modified by the correlation angle, unlike $n_{\mathcal{C}}$ and $n_{\mathcal{S}}$.

4.2 Consistency Conditions

In the single field case we have more observables than parameters, and so can derive the consistency condition that the tensor-scalar ratio is equal to

$-8n_{\mathcal{T}}$. This is modified in the two-field case, as we now have

$$\mathcal{P}_{\mathcal{R}}^{*} = \mathcal{P}_{\mathcal{R}} \cos^2 \Delta_N. \tag{58}$$

This means our modified consistency condition is

$$\frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = -8n_{\mathcal{T}} \cos^2 \Delta_N. \tag{59}$$

These quantities are all, in principle, observable. However at present for r we only have the bound $r < 0.11$ [21], and even if tensor perturbations are found, it will still take much work to establish their scale dependence.

In the single field case the upper bound on r provides an upper bound on ϵ at horizon exit, through $r = 16\epsilon$. In the multifield case, in the absence of measured isocurvature correlations, we only have an upper bound on the product $16\epsilon \cos^2 \Delta_N$.

As discussed in [7] we can obtain more consistency relations by going to higher orders in slow-roll. There are more slow-roll parameters, corresponding to third derivatives of the potential; but there are also more (albeit extremely difficult to measure) observational possibilities in the scale dependence of the spectral indices.

4.3 *Planck* 2015 Results

When the perturbations are purely adiabatic, we have that $\frac{\delta\rho}{\rho'}$ is the same for any matter component. In [22] the authors analysed temperature and polarization anisotropies in the CMB, aiming (among other things) to constrain deviations from this adiabaticity condition. For example, they looked at models where these anisotropies are partly generated by isocurvature perturbations in either cold dark matter or neutrinos.

Across the different models studied, the magnitude of the correlation angle $\sin \Delta_N$ was found to be less than ~ 0.3. This implies further suppression of the field curvature contribution to $n_{\mathcal{R}}$ (which was already suppressed by the slow-roll parameter).

Further, the isocurvature fraction

$$\beta_{iso}(k) = \frac{\mathcal{P}_{\mathcal{S}}(k)}{\mathcal{P}_{\mathcal{R}}(k) + \mathcal{P}_{\mathcal{S}}(k)} \tag{60}$$

was investigated for various wavelengths. The authors found that at large wavelengths β_{iso} could be constrained to a few percent for cold dark matter, but could be up to $\sim 20\%$ for neutrinos.

From these results we can see that so far there have been no detections of isocurvature perturbations in the CMB. This is not evidence against the generation of such perturbations during inflation, as it is entirely possible they simply did not survive reheating; however any detection would rule out single-field inflation, so they are certainly worth seeking.

5 Non-Gaussianity

One possible route to distinguishing single-field from multi-field inflation is by examining deviations from Gaussianity. The constraints on non-Gaussianity from single-field inflation are well known; naturally, the situation with two relevant fields is more complicated, and more interesting. As opposed to the power spectrum, the bispectrum examines three scales which can leave the horizon at different times. Super-horizon curvature evolution could generate large non-Gaussianity; if this non-Gaussianity is larger than the observed constraints we can rule out that model.

5.1 Known Results

One limitation of explorations of this topic is that if the curvature perturbation is still evolving at the end of inflation the observables will depend on the precise details of reheating; the theory is no longer predictive without such a prescription. Most authors have therefore only considered models where the isocurvature modes have decayed by the end of inflation.

In addition, typically only canonical kinetic terms have been considered. As ϵ measures deviations from exponential expansion, jumps in ϵ and the other slow-roll parameters have been found to generate non-Gaussianity; from (33) we can see that it is exactly then that any curvature in the field manifold becomes relevant.

While for simple attractor potentials non-Gaussianity tends to decay before it can have observable effects [23], it has been shown [24] that the situation is more hopeful for potentials with divergent trajectories, where the transfer funcitons are very sensitive to initial conditions. Interestingly, the excessive fine tuning sometimes needed to produce this non-Gaussianity can force the spectral index to violate *Planck* constraints, as found in [25] for sum potentials. It was also shown in [24] that the non-Gaussian parameters obey a new consistency relation mediated by the cross-correlation.

5.2 δN Formalism

Consider the integrated expansion between a flat hypersurface and one of uniform density; the δN formalism (e.g. [11]) tells us that the curvature perturbation generated during this evolution is the difference in the perturbed and unperturbed integrated expansions. This means that given N as a function of the fields, we can expand:

$$\mathcal{R} = \delta N = N_i \delta\phi^i + \frac{1}{2} N_{ij} \delta\phi^i \delta\phi^j + ... \tag{61}$$

This allows us to calculate the spectra of $\mathcal{R}$ in terms of the spectra of $\delta\phi^I$. For example, for the two-point function we obtain

$$\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\rangle = N_i N_j \left\langle \delta\phi^i(\mathbf{k}_1)\delta\phi^j(\mathbf{k}_2)\right\rangle \tag{62}$$

if we assume $\delta\phi^i$ is Gaussian. Yet despite assuming $\delta\phi^i$ is Gaussian we can see that $\mathcal{R}$ won't be, as its bispectrum will include terms containing the four-point function of $\delta\phi^i$, which will not vanish.

We can parametrise deviations from Gaussianity using the parameter f_{NL}, defined in [26] as

$$f_{NL} = \frac{5}{6}\frac{k_1^3 k_2^3 k_3^3}{k_1^3 + k_2^3 + k_3^3}\frac{B_{\mathcal{R}}(k_1, k_2, k_3)}{4\pi^4 \mathcal{P}_{\mathcal{R}}^2} \tag{63}$$

in terms of the bispectrum and power spectrum, enabling their comparison. Using the δN formalism one can calculate this parameter in terms of the derivatives of N [3]:

$$f_{NL} = \frac{5}{6}\frac{N_{ij}N^i N^j}{(N_i N^i)^2}. \tag{64}$$

To illustrate the use of this, we take an example, the potential[6] $V(\phi, \chi) = \frac{1}{2}g\phi^2\chi^2$. We will find that if we assume canonical kinetic terms and slow-roll right up to the end of inflation this potential cannot generate observable non-Gaussianity. During slow-roll, the equations of motion and Friedmann equation are

$$3H^2\phi' = -g\phi\chi^2, \tag{65}$$

$$3H^2\chi' = -g\chi\phi^2, \tag{66}$$

$$3H^2 = \frac{1}{2}g\phi^2\chi^2. \tag{67}$$

Taking the ratio of the equations of motion we observe that $\phi^2 - \chi^2$ is a constant for the background slow-roll trajectories. Assuming the perturbations in ϕ and χ are Gaussian at horizon exit, we can then use the above expression for f_{NL} with N a function of only ϕ, with the χ dependence implicit through the choice of trajectory. Dividing the ϕ equation of motion by the Friedmann equation we get

$$\begin{aligned} \phi' &= \frac{-2}{\phi} \\ \implies \frac{dN}{d\phi} &= -\frac{1}{2}\phi \\ \implies N(\phi) &= -\frac{1}{4}\left(\phi^2 - \phi_*^2\right) \end{aligned} \tag{68}$$

[6] This example is considered more generally in [26]

where ϕ_* is ϕ just after horizon exit. Substituting into (64) we get

$$
\begin{aligned}
f_{NL} &= -\frac{5}{3}\frac{1}{\phi^2} \\
&= -\frac{5}{3}\frac{1}{\phi_*^2 - 4N}
\end{aligned}
\tag{69}
$$

and so the increase in N during inflation suppresses non-Gaussianity.

5.3 Curvaton Scenario

We will now outline an example which can generate large non-Gaussianity. For single-field models, the field that drives inflation must also be responsible for generating the primordial density perturbation. In two-field models that restraint can be lifted; an example of this is the curvaton scenario [27]. Here we have a pair of fields. The first (called the inflaton) is assumed to drive inflation; it will not be used to generate the primordial density perturbation and is therefore not necessarily slowly rolling. The idea is that the second field χ (the curvaton) has potential $V = \frac{1}{2}m^2\chi^2$ with mass m small enough to be heavily damped during inflation. Once inflation ends, the inflaton decays into radiation, which decays as a^{-4}. Eventually H decreases to the point where the curvaton has equation of motion

$$\ddot{\chi} + m^2\chi = 0 \tag{70}$$

i.e. it begins to oscillate. This means that its pressure

$$P = \frac{1}{2}\dot{\chi}^2 - \frac{1}{2}m^2\chi^2 \tag{71}$$

averages to zero. Therefore it behaves like matter, decaying at a^{-3} and eventually dominating the radiation left over from the inflaton. It is the decay of this field, when H has decreased to the order of the decay rate of χ (which we call Γ), that seeds the density fluctuations.

The evolution thus proceeds in two phases; from horizon exit to the beginning of oscillations at $H \sim m$ (quantites denoted with a subscript m), and from then to the decay of χ at $H \sim \Gamma$ (quantites denoted with a subscript Γ). To use (64) we must determine N as a function of the fields. Following [28] we can think of N as a function of the total energy density at t_m and t_Γ as well as the horizon exit field values χ_*. To use the δN formalism our final hypersurface must be of uniform density so ρ_Γ is unperturbed. ρ_m is also uniform as it is still radiation dominated, and we assume any perturbations there are irrelevant. All perturbations in N therefore appear during the second phase, so we define N to fit the following. Since χ decays like matter in the second phase

$$\begin{aligned} N &= \ln a_\Gamma - \ln a_m \\ &= \frac{1}{3}\ln\left(\frac{\rho_{\chi m}}{\rho_{\chi\Gamma}}\right) \end{aligned} \tag{72}$$

where $\rho_{\chi m} = \frac{1}{2}m^2\chi^2(\chi_*)$ since χ has not yet begun to evolve.

We need to evaluate the dependence of $\rho_{\chi\Gamma}$ on χ_*. Since the radiation decays as a^{-4} and dominates at t_m

$$\left(\frac{\rho_\Gamma - \rho_{\chi\Gamma}}{\rho_m}\right)^{\frac{1}{4}} = \left(\frac{\rho_{\chi\Gamma}}{\rho_{\chi m}}\right)^{\frac{1}{3}} \tag{73}$$

Differentiating with respect to χ_* (denoted by $'$) we get

$$(\ln\rho_{\chi\Gamma})' = (\ln\rho_{\chi m})'\left(\frac{4\rho_\Gamma - 4\rho_{\chi\Gamma}}{4\rho_\Gamma - \rho_{\chi\Gamma}}\right) \tag{74}$$

which gives us

$$N' = \frac{1}{3}(\ln\rho_{\chi m})'\left(\frac{3\rho_{\chi\Gamma}}{4\rho_\Gamma - \rho_{\chi\Gamma}}\right) \tag{75}$$

Taking the definitions

$$R \equiv \frac{3\rho_{\chi\Gamma}}{4\rho_\Gamma - \rho_{\chi\Gamma}}, \tag{76}$$

$$y \equiv \rho_{\chi m}, \tag{77}$$

we get $f_{NL} = \frac{5}{6}\left(-2 - R + \frac{3}{R}\frac{y''y}{(y')^2}\right)$. Since $y \propto \chi^2$, we can rewrite this as

$$f_{NL} = \frac{5}{6}\left(-2 - R + \frac{3}{2R}\left(1 + \frac{\chi''\chi}{(\chi')^2}\right)\right). \tag{78}$$

Therefore if the energy density in χ is sufficiently small when it decays then f_{NL} may be large enough that the bispectrum is comparable to the power spectrum.

6 Conclusion

In this essay we explored the consequences of supposing inflation was driven by two scalar fields. Following [1, 2] we derived the equations governing the evolution of the perturbations in a form which clarifies the contributions of the slow-roll and slow-turn parameters. We also discussed the contributions of the background trajectory in the field manifold, finding multi-field effects to be important when this trajectory turned. More precisely, turns generate isocurvature modes which cause the curvature perturbation to evolve even at super-horizon scales.

Using the separate universe assumption we understood this in terms of the integrated expansion. For adiabatic perturbations the same equation of state holds everywhere, so at large scales different regions all follow the same FRW trajectory slightely displaced in time; no regions expand more or less than any others. For isocurvature perturbations however the equation of state varies, so different regions follow different trajectories; they experience different amounts of integrated expansion between uniform density hypersurfaces, and therefore the curvature perturbations evolve.

This presents a serious difficulty in making predictions. We do not understand the era of reheating; this matters less in single-field inflation because the perturbations are frozen, but here we have no such luxury. Most authors proceed by constraining themselves to the case of isocurvature perturbations that decay before the end of inflation.

Some predictions are robust though; from (59) we can see that even if we cannot measure the correlation angle, it is a necessary prediction of two-field slow-roll inflation that the scalar-tensor ratio is bounded by $-8n_{\mathcal{T}}$. We also saw that $n_{\mathcal{S}}$ is less sensitive to reheating than $n_{\mathcal{R}}$, and the interesting possibility that the cross-correlations could have significant scale dependence.

One other difficulty with two-field inflation is the weighting of initial conditions. This is a generic problem in cosmology though; it is hard to rule out possibilities when their initial conditions leave room for fine-tuning. The work of [25] is interesting in that respect; using experimental bounds to constrain fine-tuning possibilities, and so place testable bounds on other observables.

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